Core 3 - January 2006

(1) a) $y = \tan(3x)$

chain rule

$dy/dx = 3\sec^2(3x)$

b) $y = \dfrac{3x+1}{2x+1}$ $\qquad u = 3x+1 \qquad v = 2x+1$

quotient rule

$du/dx = 3 \qquad dv/dx = 2$

$$dy/dx = \frac{3(2x+1) - 2(3x+1)}{(2x+1)^2}$$

$$= \frac{6x+3-6x-2}{(2x+1)^2} = \frac{1}{(2x+1)^2}$$

(2)

x	1	1.5	2	2.5	3
y	0.7071	0.4781	0.3333	0.2453	0.1890

$h = 0.5$

$$\text{Area} = 1/3 \times 0.5 \times [0.7071 + 0.1890 + 4(0.4781 + 0.2453) + 2(0.3333)]$$

$$= 0.743 \ (3sf)$$

(3) a) i) $f(x) = x^4 + 2x$

$f'(x) = 4x^3 + 2$

ii) $\displaystyle\int \frac{2x^3+1}{x^4+2x} = 1/2 \int \frac{4x^3+2}{x^4+2x} = 1/2 \ln(x^4+2x)$

b) i) $\displaystyle\int x\sqrt{2x+1}\,dx$ $\qquad u = 2x+1$

$du/dx = 2 \rightarrow dx = 1/2\,du$

$\displaystyle\int x\,u^{1/2}\,1/2\,du$

$u = 2x+1$

$u - 1 = 2x$

$\rightarrow x = \dfrac{u-1}{2}$

$$\int \left(\frac{u-1}{2}\right)\left(u^{1/2}\right) 1/2\,du$$

$$= 1/4 \int (u-1)\,u^{1/2}\,du$$

$$= 1/4 \int u^{3/2} - u^{1/2}\,du$$

ii) $\displaystyle\int_0^4 x\sqrt{2x+1}\,dx$

change limits $\quad u = 2x+1$

$$= 1/4 \int_1^9 u^{3/2} - u^{1/2}\,du$$

$(x=0) \quad u = 1$

$(x=4) \quad u = 9$

$$= \tfrac{1}{4}\left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right]_1^9$$

$$= \tfrac{1}{4}\left[\frac{9^{5/2}}{5/2} - \frac{9^{3/2}}{3/2} - \frac{1^{5/2}}{5/2} + \frac{1^{3/2}}{3/2}\right]$$

$$= \tfrac{1}{4} \times 79.46666 = 19.8666... = 19.9 \ (3sf)$$

(4) $2\operatorname{cosec}^2(x) = 5 - 5\cot(x)$

a) $\operatorname{cosec}^2(x) = 1 + \cot^2(x)$

$\rightarrow 2(1+\cot^2(x)) = 5 - 5\cot(x)$

$2 + 2\cot^2(x) = 5 - 5\cot(x)$

$\rightarrow 2\cot^2(x) + 5\cot(x) - 3 = 0$

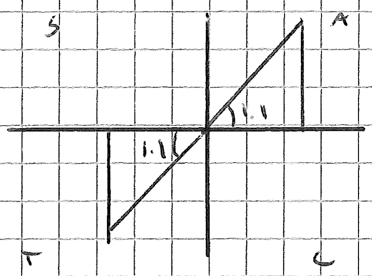
b) $(2\cot(x) - 1)(\cot(x) + 3) = 0$

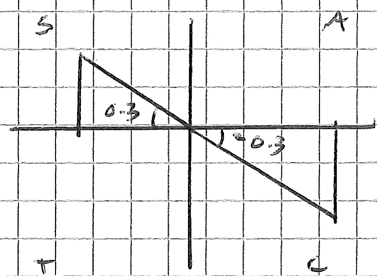
$\downarrow$ $\qquad\qquad\qquad \downarrow$

$\cot(x) = 1/2$ $\qquad\qquad \cot(x) = -3$

$\rightarrow \tan(x) = 2$ $\qquad\qquad \rightarrow \tan(x) = -1/3$

c) $x = 1.107...$ $\qquad\qquad x = -0.3217...$





$x = 1.107..$ $\qquad\qquad x = -0.3217$

$x = -2.034...$ $\qquad\qquad x = 2.8198...$

$x = 1.1, -2.0, -0.3, 2.8 \quad (1dp)$

(5) a) $y = e^{2x} - 9$

(a) $x = 0 \rightarrow y = 1 - 9 = -8 = a$

(b) $y = e^{2x} - 9$ when $y = 0$

$e^{2x} - 9 = 0$

$e^{2x} = 9$

$2x = \ln(9) \rightarrow x = \tfrac{1}{2}\ln(9) \rightarrow x = \ln(\sqrt{9}) = \ln(3)$

b) $y^2 = (e^{2x} - 9)^2 = e^{4x} - 9e^{2x} - 9e^{2x} + 81$

$= e^{4x} - 18e^{2x} + 81$

c) $V = \pi \int_0^{\ln(3)} y^2 \, dx$

$= \pi \int e^{4x} - 18e^{2x} + 81 \, dx$

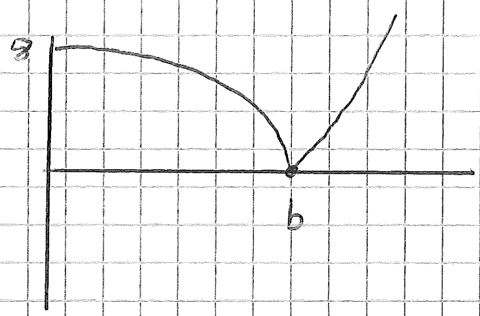
$= \pi \left[\frac{1}{4} e^{4x} - 9e^{2x} + 81x \right]_0^{\ln(3)}$

$= \pi \left[\frac{1}{4} e^{4\ln(3)} - 9e^{2\ln(3)} + 81\ln(3) - \frac{1}{4}e^0 + 9e^0 + 81(0) \right]$

$= \pi \left[\frac{81}{4} - 81 + 81\ln(3) - \frac{1}{4} + 9 + 0 \right]$

$= \pi \left[-52 + 81\ln(3) \right] = \pi(81\ln(3) - 52)$

d)



⑥ a) $f(x) = x^3 + 4x - 3$

$f(0.5) = 0.5^3 + 4(0.5) - 3 = -\frac{7}{8}$

$f(1) = 1^3 + 4(1) - 3 = 2$

Sign change, therefore root lies between 0.5 and 1

b) $x^3 + 4x - 3 = 0$

$4x = 3 - x^3$

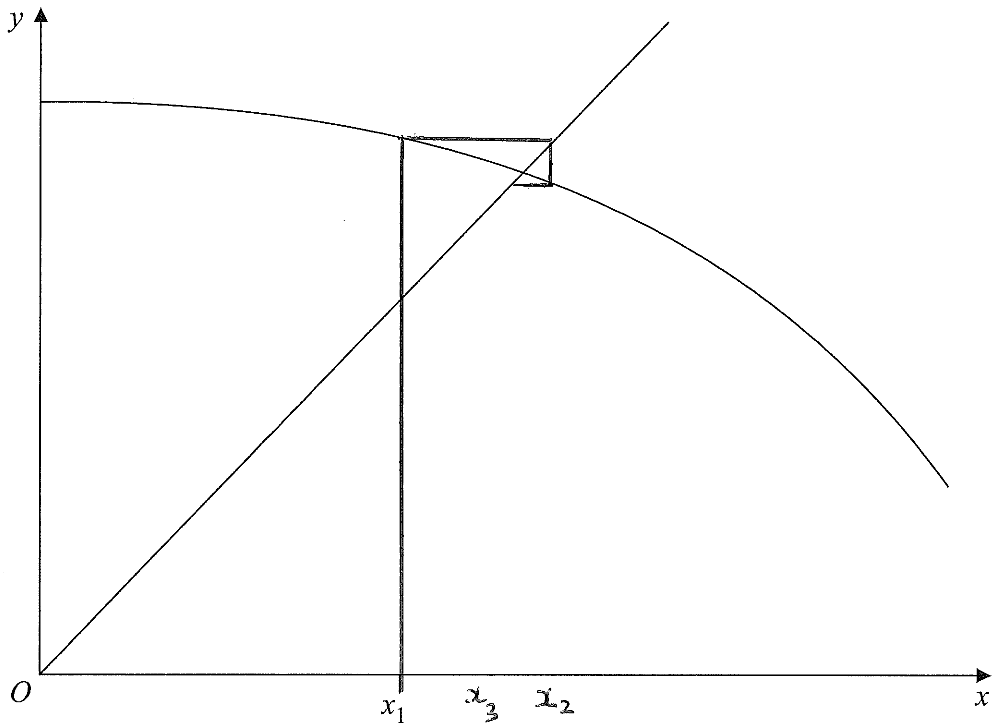
$x = \dfrac{3 - x^3}{4}$

c) $x_1 = 0.5$

$x_2 = \dfrac{3 - 0.5^3}{4} = 0.71875$

$x_3 = \dfrac{3 - 0.71875^3}{4} = 0.65713\ldots = 0.66$ (2dp)

Figure 1 (for Question 6)



9 a) $y = x^{-2}\ln(x)$ $u = x^{-2}$ $v = \ln(x)$

$du/dx = -2x^{-3}$ $dv/dx = 1/x$

PRODUCT RULE

$dy/dx = -2x^{-3}\ln(x) + x^{-2} \times 1/x$

$= \dfrac{-2\ln(x)}{x^3} + \dfrac{1}{x^3} = \dfrac{1 - 2\ln(x)}{x^3}$

b) $\int x^{-2}\ln(x)$ $u = \ln(x)$ $dv/dx = x^{-2}$

$du/dx = 1/x$ $v = -x^{-1}$

$= -x^{-1}\ln(x) - \int -x^{-1} \times \dfrac{1}{x}\, dx$

$= -\dfrac{1}{x}\ln(x) - \int -x^{-2}\, dx$

$= -1/x\ \ln(x) - [+x^{-1}]$

$= -1/x\ \ln(x) - 1/x + c$

c) i) at st point, $dy/dx = 0$

$\dfrac{1 - 2\ln(x)}{x^3} = 0$

$\Rightarrow 1 - 2\ln(x) = 0$

$1 = 2\ln(x)$

$1/2 = \ln(x)$

$e^{1/2} = x$

ii) from b) $\left[-1/x\ \ln(x) - 1/x\right]_1^5$

$= -1/5\ln(5) - 1/5 + \ln(1) + 1$

$= -1/5\ln(5) + 4/5$

$= 1/5(4 - \ln(5))$